

Course Title: Group Theory

Course Code: MATH-206

Course Type: Major Math

Prerequisites: N/A

Credit Hours: 3 (3 + 0)

Course Objectives: By the end of the course students should be able to:

- Gain a thorough understanding of the fundamental concepts and various applications of groups.
- Provide basic concepts of group theory including cyclic groups, normal subgroups, group homomorphism.
- Enhance their capacity for mathematical reasoning, develop skills in constructing simple proofs, and cultivate the ability to critically evaluate the correctness and completeness of proofs within the domain of group theory.

Course Contents:

Preliminaries: Relations, Mappings, Binary operation, Groupoid, Semigroup,

Introduction to Groups: Definition of a Group and its examples, Elementary properties of groups, Abelian groups, Cyclic groups, Dihedral groups, Quaternion groups, Matrix groups, Group of integers modulo n \mathbb{Z}_n .

Subgroups: Definition of subgroup, Properties of subgroups, Cosets and Lagrange's Theorem, Centralizer of an element of a group, Centre of a group, Normalizer of a subset in a group, Commutator subgroup of a group.

Normal Subgroups and Factor Groups: Definition of normal subgroup, Characterization of normal subgroups, Factor groups, Simple groups, Direct product of two groups and examples.

Group Homomorphisms: Definition of group homomorphism, Kernel of a homomorphism, Properties of homomorphisms, Isomorphism theorems, Correspondence theorem, Automorphisms of a group, Conjugation, Conjugacy classes of groups.

Permutation Groups: Definition of permutation group, Cycles, Symmetric Groups, Conjugacy classes of Symmetric groups and Alternating groups, Cayley's theorem.

Group Action: Group actions and its examples, Orbit-Stabilizer theorem.

Recommended Books:

1. Fraleigh, J. B., *A First Course in Abstract Algebra*, Pearson, 7th edition, 2002.
2. Gallian, C. J., *Contemporary Abstract Algebra*, Chapman and Hall/CRC, 10th edition, 2020.
3. Herstein, I. N., *Topics in Algebra*, John Wiley & Sons, 2nd edition, 1991.

4. Rotman, J., *An Introduction to the Theory of Groups*, 4th edition, Springer, 1995.
 5. Smith, G. C. and Tabachnikova, O. M., *Topics in Group Theory*, Springer, 2000.
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